**Field-based Fuzzy Spatial Reasoning Model for Geographical Information Systems: Case of Constraint Satisfaction Problem**

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**Abstract:** Humans’ representation in nature language about geographic phenomena is usually qualitative rather than quantitative. Qualitative spatial reasoning provides an approach which is considered to be closer to the representation. Commercial GIS software are confronted with a challenge that the software should be equipped with artificial intelligent functions like qualitative spatial reasoning for more and more users, especially for spatial decision-makers. This paper proposes a framework of field-based fuzzy spatial reasoning through which qualitative description usually encountered in spatial reasoning process can be handled quantitatively. As preconditioning, field-based fuzzy representation structure for qualitative description is put forward, then the methods of constructing membership function are discussed. Standard operations of field-based fuzzy spatial reasoning model in the case of constraint satisfaction problem (CSP) are illustrated. An example explains the implication of the model in spatial decision-making process.

**Keywords:** Geographical information systems, Fuzzy membership values, Qualitative spatial reasoning, Field-based models, Constraint satisfaction problem

1. **Introduction**

While geographical information systems (GIS) have attracted many attentions for its strong functions of managing spatial information and providing spatial decision-making tools to users, there has been little change in the functionality of the systems (Guesgen and Albrecht, 2000). The way in which they perform spatial reasoning, i.e., the extraction of new information from stored spatial data, has been intuitively quantitative in nature. On the other hand, humans often prefer a qualitative analysis over a quantitative one, as this is more adequate in many cases from the cognitive point of view (Clementini et al., 1997). For instance, in the sentence “to find a parcel of wasteland which is not far from a reservoir”, extension space of the word “not far” is so vague that current spatial analysis functions of GIS software face difficulty to solve analogous problems. Dealing with these problems entails qualitative spatial representation and reasoning approaches.

GIS are enormously complicated software systems which particularly process spatial knowledge. Spatial decision-making under the support of GIS can be conceived as technique extension of traditional map analysis function. As most spatial expressions in natural language are purely qualitative, the lack of function of effective qualitative spatial reasoning like Artificial Intelligence (AI) in software inevitably limits the application of GIS in the spatial decision-making process. Stefanakis et al. (1999) discussed in detail the significance of providing efficient tools in GIS packages to decision-makers who use GIS and emphasize mainly the uncertainty of geographical phenomena in GIS.

Two options are available for extending the functions of GIS: to extend the functions of spatial analysis of GIS software repository with qualitative spatial reasoning and to implement qualitative spatial analysis (reasoning) functions in special application systems associated with GIS through extension programming. But both of the options must face the difficulties of describing the uncertainty, especially fuzzy uncertainty, of real-life geographical phenomena and ambiguity of human language. All the confronted problems influence the implementation of qualitative spatial reasoning function in GIS. Former difficulty mainly affects the representation.
and store of spatial knowledge. For examples, the boundary between woodland and grassland, and between urban and rural areas, may be gradual through a transition zone rather than a crisp boundary. Some researchers attempt to find alternative approaches to represent the type of phenomena (Molenaar and Cheng, 2000; Cheng et al., 2001) and topological relations of these special objects (Bjørke, 2004), whereas the approaches have yet not been effectively embedded in current commercial GIS software. The latter is more complex as it is difficult to model human natural language which usually contains vague instructions. Laudably, some literatures in other disciplines are looking at the problems (Bloch and Ralescu, 2003; Claramunt and Thériault, 2004; Renz, 2002). Fuzzy sets show superiority in representing qualitative phenomena. This paper proposes a framework of field-based spatial reasoning using fuzzy set theory through which qualitative description usually encountered in spatial analysis function can be handled quantitatively.

The remainder of the paper is organized as follows. The next section reviews the definition of fuzzy sets and options of fuzzy membership functions. Section 3 discusses object-based and field-based data models applied to represent spatial entities in GIS, and put forward field-based model for qualitative spatial knowledge. This is followed by proposing field-based fuzzy spatial reasoning models in the case of constraints satisfaction problems. Then a case study compares the approach with traditional spatial reasoning process. Sections 6 and 7 give some discussions and summarize the results.

2. Fuzzy sets

2.1. Fuzzy set theory

In classical set theory (Boolean logic) an individual is a member or not a member of any given set. The membership degree to which the individual \( z \) belongs to the set \( A \) is expressed by the membership function, \( \mu_A \), which can take the value 0 or 1, i.e.,

\[
\mu_A(z) = \begin{cases} 
1, & c_1 \leq z \leq c_2 \\
0, & z < c_1 \text{ or } z > c_2 
\end{cases} 
\tag{1}
\]

Where, \( c_1 \) and \( c_2 \) define the boundaries of set \( A \). For example, if the boundaries between the direction “north”, “east”, “south” from object \( O_1 \) to object \( O_2 \) were to be set at azimuth \( \theta_1=45^\circ \) and \( \theta_2=135^\circ \) from \( O_1 \) to \( O_2 \) (Figure 1), then the membership function defines the direction “east” (Figure 2a). Note that classical sets allow only binary membership functions (i.e., TRUE of FALSE).

![Figure 1 Definition of azimuth \( \theta \) from object \( O_1 \) to \( O_2 \).](image1.png)

![Figure 2 An example of the classic (a) and fuzzy (b) classification for direction. \( \theta \) stands for the azimuth from object \( O_1 \) to \( O_2 \).](image2.png)

Fuzzy set theory (Zadeh, 1965) is an extension of classical set theory. A fuzzy set \( A \) in a universe \( Z \) of discourse is characterized by a membership function \( \mu_A(z) \) which associated with a real number in the interval \([0,1]\), representing the “degree of membership of \( z \) in \( A \)”. Thus, the nearer the value of \( \mu_A(z) \) to unity, the higher the grade of membership of \( z \) in \( A \). That is to say, \( \mu_A(z) \) of \( z \) in \( A \)
specifies the extent to which \( z \) can be regarded as belonging to set \( A \). The fuzzy sets can be represented as a set of ordered pairs:

\[
A = \{z, \mu_A(z)\}, \quad z \in Z
\]  

(2)

2.2. Fuzzy membership function

The choice of fuzzy membership function, i.e., its shape and form, is crucial in fuzzy sets application. In correspondence to classical set theory, two options are available for choosing the membership functions for fuzzy sets (Burrough, 1996): (a) through an imposed ‘expert’ model; and (b) by a data driven multivariate procedure.

The first approach uses a priori membership function, based on expert knowledge, with which individual entities can be assigned a degree of membership regarding a lexical value characterizing a theme. This method is known as the semantic import approach or model. Each of these functions has its own characteristics and its behavior may simulate better or worse various physical phenomena. Several conventional linear models which can be used as membership function are shown in Figure 3.

Also some nonlinear models can be applied to represent fuzzy phenomena in terms of their characteristics. In aforementioned direction example, we choose \( \sin^2(\theta) \) and \( \cos^2(\theta) \) referring to relative position relation functions proposed by Miyajima and Ralescu (1994) to describe the direction [north, east, south, west] from object \( O_1 \) to \( O_2 \) (Figure 1) as follows.

\[
\mu_{\text{north}}(\theta) = \begin{cases} 
\cos^2(\theta), & \frac{3\pi}{2} \leq \theta \leq 2\pi \text{ or } 0 \leq \theta \leq \frac{\pi}{2} \\
0, & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}
\end{cases}
\]  

(3)

\[
\mu_{\text{east}}(\theta) = \begin{cases} 
\sin^2(\theta), & 0 \leq \theta \leq \pi \\
0, & \pi \leq \theta \leq 2\pi
\end{cases}
\]  

(4)

\[
\mu_{\text{south}}(\theta) = \begin{cases} 
\cos^2(\theta), & \frac{3\pi}{2} \leq \theta \leq 2\pi \text{ or } 0 \leq \theta \leq \frac{\pi}{2} \\
0, & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}
\end{cases}
\]  

(5)

\[
\mu_{\text{west}}(\theta) = \begin{cases} 
\sin^2(\theta), & \pi \leq \theta \leq 2\pi \\
0, & 0 \leq \theta \leq \pi
\end{cases}
\]  

(6)

Where, \( \theta \) stands for the azimuth from object \( O_1 \) to \( O_2 \). Figure 2b illustrates the fuzzy classification of direction values. Note that the classic way (Figure 2a) to classify direction involves discrete classes with special ranges, while fuzzy classification captures the graduate transition between classes, providing a better way to categorize imprecise concepts, such as north and east direction. It is more “human-like” or “cognitively adequate” than the classic way.

In the second approach the choice of the membership functions is data-driven in the sense that they are locally optimized to match data set. This method is analogous to cluster analysis and numerical taxonomy (Kaufman and Rousseuw, 1990) and is known as the natural classification model.

Whichever approach is chosen, the ultimate form or shape of the function should be “human-like” and close to the reality.

3. Field-based qualitative spatial representation

3.1. Field-based models

In order to implement spatial decision-making for users, GIS have to provide the ability of representing spatial phenomena which intersperse in the space under certain data models. As pointed out by Goodchild (1992), the GIS
data models are divided into two broad categories. First, entity-based or object-based data models which represent geographic data set conceived as collections of discrete objects littering an otherwise empty space, and able to overlap freely. Second, field-based models which represent variation that is conceived as being spatially continuous, such that for every point in the plane there is exactly one value of the field.

Spatial entity refers to a phenomenon that cannot be subdivided into like units (Laurini and Thompson, 1992). A house is not divisible into houses, but can be split into rooms. The discrete object view represents the world as objects with well-defined boundaries in empty space (Longley et al., 2001). Entity-based data models catch the characteristics of categories of spatial phenomena and make it easy to represent spatial phenomena in vector data structure. Therefore, the models show superiority in representing spatial entity with crisp (well-defined) boundaries. But they are not suited to mapping poorly defined phenomena (Zhang and Stuart, 2001) and qualitative description.

Field-based data models come from field view of representing geography. For example, while we might think of terrain as composed of discrete mountain peaks, valleys, ridges, slopes, etc., there are unresolvable problems of definition for all of these objects. Instead, it is much more useful to think of terrain as a continuous surface, in which elevation can be defined rigorously at every point. Such continuous surfaces form the basis of the common view of geographic phenomena, known as the field view. In this view the geographic world can be described by a number of variables, each measurable at any point on the Earth’s surface, and changing in value across the surface (Longley et al., 2001).

In digital representation, field-based data models can be represented as the following continuous two-order relationship on 2-D plane $N^2$ (Zhao et al., 1999):

$$ R = \int\int_{(x,y)} \frac{\mu_R(x,y)}{(x,y)} \quad x, y \in N^2 \quad (7) $$

Where, fuzzy membership value $\mu_R(x, y)$ represents attribute density of a surface feature character on the point $(x, y)$. That is to say, it stands for the extent to which a point belongs to one class (object). If $\mu_R(x, y)$ equals anyone of both numbers $\{0, 1\}$, it indicates that all the objects in real-life have crisp boundaries. If $\mu_R(x, y)$ is a numerical value in the interval $[0, 1]$, $R$ becomes a fuzzy sets (Zadeh, 1965) and the model can represent fuzzy geographical phenomena.

The relationship can be expressed with a 2-D matrix in which row and column numbers are the coordinates of the spatial surface feature. For example, urban area can be represented as following Figure 4:

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0.1 0.1 0.1 0.1 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.0 1.0 1.0 1.0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.0 1.0 1.0 1.0 1.0
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.0 1.0 1.0 1.0 1.0 1.0
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0
0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0
0.5 0.6 0.7 0.8 0.9 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0
0.6 0.7 0.8 0.9 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0
```

Figure 4 Fuzzy representation of spatial object - urban area. (a) in numerical form; (b) in grey form. Grey values hierarchy is shown in the right of (b).

The value of cells in Figure 4 stands for the extent to which the cell belongs to urban area. 1.0 indicates that the cell belongs to the classification of urban area entirely; 0 < the value <1.0 means that the cell belongs to the urban area partly; 0, the cell does not be characterized by urban area at all. The two-order relationship reflects the field view of geographic phenomena.

3.2. Field-based qualitative spatial representation

In geographical space, reasoning on spatial entities is
supported by natural language representations that involve direction, topological, ordinal, distance, size and shape relationships (Pullar and Egenhofer, 1988). In qualitative spatial reasoning it is common to consider the main spatial aspects: topology, direction, distance and to develop a system of qualitative relationships between spatial entities which cover this spatial aspect to some degree and which appear to be useful from an applicational or from a cognitive perspective (Renz, 2002). Many traditional commercial GIS soft wares include the functions of representing topological relationships of spatial entities. Therefore in this study we just consider the representation of other two important spatial aspects: direction and distance.

When we describe the relationships of spatial entities we always select one of spatial entities as a reference object. The relationships of other spatial entities with the reference object are associated with the locations of them. That is to say, the relationships can be considered as a field surrounding the reference object. Therefore, the spatial aspects also can be represented using the field-based models as continuous two-order relationship on 2-D plane $N^2$ (see formula 7). Here, the fuzzy membership value $\mu_{R(x, y)} \in [0, 1]$ represents the degree to which the point $(x, y)$ belongs to the relationship representation of between spatial entities according to their location. We name the approach as field-based qualitative spatial representation structure.

3.2.1. Representation of direction

Direction—also called orientation—relationships of spatial entities with respect to other spatial entities is usually given in terms of a qualitative category like “to the north of” rather than using a numerical expression like “12 degrees” (which is certainly more common in technical communication like in aviation). It is important and common-sense linguistic and qualitative property used in everyday situations and qualitative spatial reasoning (Frank, 1996). Direction of spatial entities is a ternary relationship depending on the located object, the reference object, and the frame of reference which can be specified either by a third object or by a given direction. In the literature one distinguishes between three different kinds of frames of reference, extrinsic (“external factors impose a direction on the reference object”), intrinsic (“the direction is given by some inherent property of the reference object”), and deictic (“the direction is imposed by the point of view from which the reference object is seen”) (Hernández, 1994). Given the frame of reference, direction can be expressed in terms of binary relationships with respect to the frame.

Most approaches to qualitatively dealing with direction are based on points as the basic spatial entities and consider only two-dimensional space. Frank (1991) suggested different methods for describing the cardinal direction of a point with respect to a reference point in a geographic space, i.e., directions are in the form of [north, east, south, west] depending on the granularity. Without loss of generality, here we consider all objects even that with irregular shape and size as a point and follow frank’s suggestion about direction (centroid-based method, where the direction between two objects is determined by the angle between their centroids). According to Figure 1, the azimuth $\theta$ from object $O_1$ to object $O_2$ is computed. This angle, denoted by $\theta(O_1, O_2)$, takes values in $[0, 2\pi]$, which constitutes the domain on which primitive directional relations are defined.

We choose $\sin^2(\theta)$ and $\cos^2(\theta)$ as fuzzy membership functions to describe the direction [north, east, south, west] referring to relative position relation functions proposed by Miyajima and Ralescu (1994). See formulae 3, 4, 5, and 6 and Figure 2b. Miyajima and Ralescu (1994) have used the square trigonometric function to illustrate relative position relations [above, right, below, right] of segmented images. The square trigonometric functions are also suitable for the direction in the form of [north, east, south, west]. For instance, in Figure 1, if $\theta = 50^\circ$, then the direction relationship is $[0.4132, 0.5868, 0, 0]$ in the form of [north, east, south, west] according to formulae 3, 4, 5, and 6. It means that object $O_2$ is located to the north of object $O_1$ with 0.4132 of membership degree and to the east with 0.5868 of membership degree. That is, $\mu_{north}(O_1, O_2)= 0.4132$, $\mu_{east}(O_1, O_2)= 0.5868$, $\mu_{north}(O_1, O_2)= 0$, $\mu_{east}(O_1, O_2)= 0$, and $\mu_{north}(O_1, O_2)+ \mu_{east}(O_1, O_2)= 1$. Therefore, the fuzzy membership functions not only show the characteristics of transition of
direction relationship but also ensure the integrality of definition of direction for any target object.

In GIS, the direction relations can be seen as field view in order to represent them. An example is given to illustrate the representation of direction relations using field-based qualitative spatial representation structure. In the example, we require representing the linguistic sentence “to the north of a city”. According to formula 3, we get the field-based representation in raster structure like Figure 5a as numerical form and Figure 5b as grey form. Here, the city is abstracted as a point.

![Figure 5](a) Field-based representation of “to the north of city C”. (a) in numerical form; (b) in grey form. Grey values hierarchy is shown in the right of (b).

In Figure 5, the marked point with “C” stands for the city. The number in Figure 5a or the grey grade in Figure 5b denotes the extent to which the location belongs to the linguistic sentence “to the north of a city”.

### 3.2.2. Representation of distance

In spatial decision-making process, distance relation between spatial entities always plays a key role. Dealing with distance information is an important cognitive ability in our everyday life (Renz, 2002). When representing distance, we usually use qualitative categories like “A is close to B” (binary constraint) or qualitative distance comparatives like “A is closer to B than to C” (ternary constraint), but also numerical values like “A is about 20 m away from B”. One can distinguish between absolute distance relations (the distance between two spatial entities) and relative distance relations (the distance between two spatial entities as compared to the distance to a third entity) (Guesgen and Albrecht, 2000; Renz, 2002). The choice which relations should be used depends on the application domain and the requirements posed by decision-makers. In this work we restrict ourselves to absolute distance relations, i.e., binary constraints. For two individual locations A, B, which are abstracted as points in general, the Euclidean distance is given by the formula:

\[
d(A, B) = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}
\]

(8)

Where \((x_A, y_A)\) and \((x_B, y_B)\) denote the coordinates of two locations A, B, respectively.

Qualitative absolute distance relations are obtained, e.g., by dividing the real line of distance into several sectors such as “very close”, “close”, “commensurate”, “far”, and “very far” depending on the chosen level granularity (Hernández et al., 1995). In practice we usually use one of the sectors. For instance, Figure 6 represents the “close” degree from a point on the map to a city.

![Figure 6](b) Membership function of “close to a city”.

Where, \(\mu_{\text{close}}(x)\) denotes the distance (in kilometer) from a location to the city. The division values such as 5km and 20km are
designed arbitrarily by decision-makers according to the understanding of definition of “close” degree.

In field-based model, distance relations are represented as a field surrounding the reference spatial entity. Every location in the field has a membership value that means the extent to which the location belongs to the qualitative absolute distance relations. In the following we use field-based representation of above example to illustrate the model (Figure 7). Where, the marked point with “C” stands for the city. The number in Figure 7a or the grey grade in Figure 7b denotes the grade of fuzzy membership, the extent to which the location belongs to the linguistic sentence “close to a city”.

![Field-based representation of “close to a city C”](image)

4. Field-based fuzzy spatial reasoning: case of constraint satisfaction problem

Spatial qualitative reasoning is an approach for dealing with commonsense spatial knowledge. While knowledge about spatial entities or about the relationships between spatial entities are often given in the form of constraints. Ordinarily binary constraints like “the primary school should be laid out in the north of the residential area”, ternary constraints like “the primary school should be laid out between residential area A and residential area B”, or in general n-ary constraints restrict the domain of 2, 3, or n variables. Problem like these are formalized as a constraint satisfaction problem (CSP): given a set of variables $R$ over a domain $D$ and a set $A$ of constraints on the variables $R$ (Renz, 2002). The constraint satisfaction problem is a powerful general framework in which a variety of combinatorial problems can be expressed (Creignou et al., 2001; Marriott and Stuckey, 1998). The aim in a constraint satisfaction problem is to find an assignment of values to the variables subject to specified constraints. In fact, it is the most popular reasoning methods used in qualitative spatial reasoning (Renz, 2002) and a common problem in spatial decision-making process like example above. In this research we restrict to binary CPSs, i.e., CSPs where only binary constraints are used.

In order to deal with the problem, Ladkin and Maddux (1994) formulated binary CPSs as relation algebras developed by Tarski (1941). This allows treating binary CPSs in uniform way. In fuzzy domain, the relation algebras constitute fuzzy logic reasoning. Fuzzy logic reasoning, one of application domains of fuzzy relationship generalization, is the fundamental of fuzzy spatial reasoning. It implements tasks through logical operations based on usual relation algebra theory. The standard operations of union, intersection, and complement of fuzzy relationship (or fuzzy sets) $A_1$ and $A_2$, defined over some domain $C$, create a new fuzzy relationship (or fuzzy sets) whose membership function is defined as:

**Union:**

$$\mu_B(z) = \mu_{A_1 \cup A_2}(z) = \max\{\mu_{A_1}(z), \mu_{A_2}(z)\}, z \in C \quad (10)$$

**Intersection:**

$$\mu_B(z) = \mu_{A_1 \cap A_2}(z) = \min\{\mu_{A_1}(z), \mu_{A_2}(z)\}, z \in C \quad (11)$$

**Complement:**

$$\mu_B(z) = \mu_{A_1'}(z) = 1 - \mu_{A_1}(z), z \in C \quad (12)$$

Figure 8 illustrates the significance of the operations intuitively.
The operations also can be extended to \( n \) sets of fuzzy relationships, that is, the operation is applicable to multiple fuzzy sets. Assume that there are \( n \) sets of fuzzy relationships, operations above can be expressed uniformly as:

\[
\mu_{R}(z) = \bigotimes_{i=1}^{n} \mu_{A_{i}}(z) \quad z \in \mathbb{C} \quad (13)
\]

Where, \( \bigotimes \) denotes operators union, intersection, and complement respectively, and \( A_{i} \) stands for multiple fuzzy relationships. The results of operations stand for the suitableness of satisfying CSPs.

\[\mu(x)\]

\[\mu(x)\]

\[\mu(x)\]

\[\mu(x)\]

\[\mu(x)\]

**Figure 8** Diagram of fuzzy spatial reasoning operations.

(a) Fuzzy membership function of \( A_{1} \). (b) Fuzzy membership function of \( A_{2} \). (c) Union of \( A_{1} \) and \( A_{2} \). (d) Intersection of \( A_{1} \) and \( A_{2} \). (e) Complement of \( A_{1} \). Thick lines in (c), (d), and (e) stand for the result of operations.

5. Application to an illustrative case study

An example is given to illustrate the application of field-based fuzzy spatial reasoning model. In this example, the task is to find a suitable location for a special factory given certain constraining factors as followings:

(a) The factory must be located to the east of the environmental monitoring station.

(b) The factory must not be far from the environmental monitoring station.

(c) The factory must not be situated on land suitable for agriculture.

It is difficult to handle the task just using existing spatial analysis function of traditional GIS software as the constraining factors described above could not be transformed into numeric representation suitable for existing geographical information systems. Alternative way suitable for conventional spatial overlay analysis is to alter the description manner of the constraining factors as:

(a) The azimuth from environmental monitoring station to the factory must be between 45° and 135°.

(b) The distance from the factory to environmental monitoring station must be between 4 and 9 km.

(c) The land must not fall into agricultural category field.

\[\mu(x)\]

\[\mu(x)\]

\[\mu(x)\]

\[\mu(x)\]

\[\mu(x)\]

**Figure 9** Diagram of traditional approach process in GIS for the example. The point with a symbol of house denotes the environmental monitoring station. Satisfaction areas of constraints are expressed as black.

(a) Direction constraint. (b) Distance constraint. (c) Non agricultural area constraint. (d) The result satisfying the constraints.

In existing GIS software, this type of reasoning uses the
concepts of Boolean map overlay. This is a method for merging different datasets to produce a final output. Various functions are available, including a buffer operator, which increases the size of an object by extending its boundary, and logical operators, such as union, intersection and complement. Figure 9 illustrates the process of traditional approach in GIS. But the alternative way is often too rigid and therefore not applicable to scenarios like factory scenario. The reason is that the description like ‘between 4 and 9 km’ may eventually result in an empty map, as they restrict the search space too dramatically by excluding an areas, for example, which are 9.1 km from environmental monitoring station. Such an area, however, might be the best choice available and therefore perfectly acceptable.

It is evident that the result of alternative way in GIS has clear boundary (Figure 9d). If the agriculture area is larger than that of above example, maybe there are no areas satisfying the constraints. Now we solve the problems using field-based fuzzy spatial reasoning approach. A possible solution might be as following:

1) Determine fuzzy membership function for the description ‘to the east of the environmental monitoring station’, and constitute fuzzy relationship 

\[
A_1 = \int_{(x,y)} \frac{\mu_A(x,y)}{(x,y)}
\]

on research area by calculating membership value of cell in term of fuzzy membership function. Here we select the membership function discussed in section 3.2.1. See Figure 10a.

2) Determine fuzzy membership function for ‘not be so far from the environmental monitoring station’, and constitute fuzzy relationship 

\[
A_2 = \int_{(x,y)} \frac{\mu_A(x,y)}{(x,y)}
\]

Here we select the following function:

\[
\mu_{notsofar}(x) = \begin{cases} 
\frac{x}{4}, & x \leq 4 \\
1, & 4 < x \leq 9 \\
\frac{9-x}{6}, & 9 < x \leq 15 \\
0, & x \geq 15 
\end{cases}
\]

(14)

Where, x stands for the distance from the location to the environmental monitoring station with kilometers. See Figure 10b.

![Figure 10](image-url) Diagram of fuzzy spatial reasoning process. The point with a symbol of house denotes the environmental monitoring station. Grey stands for the satisfaction degree for the constraints. (a) Direction constraint. (b) Distance constraint. (c) Non agricultural area constraint. (d) The result satisfying the constraints.
3) Determine fuzzy membership function for ‘suitable for agriculture’, and constitute fuzzy relationship \( A_j = \int_{(x,y)} \mu_{a_j}(x,y) \). Here we select classical function to represent the constraint as follow:

\[
\mu_{\text{suit}-\text{non-a}} = \begin{cases} 
1 & \text{not in agricultural area} \\
0 & \text{in agricultural area} 
\end{cases} 
\]  

(15)

See Figure 10c.

4) Use fuzzy spatial reasoning \( R = A_1 \cup A_2 \cup A_3 \) to calculate the fuzzy membership value which stand for the degree of suitableness of different location for the proposition of the task. See Figure 10d.

It is evident that the information of the result (Figure 10d) is more abundant and more careful compared with that of result of traditional approach (Figure 9d). The approach gives decision-makers more chance to choose suitable result as it provides the degree of suitableness of the proposition proposed by the users.

6. Discussion

1) In this example, the operation used is intersection which means the constraining factors have the same weight and all of them have to be satisfied in factory scenario. If we select union as operation, it means anyone of the factors satisfied can determine the result. Therefore, operations in field-based fuzzy spatial reasoning model should be chosen in terms of the proposition.

2) After spatial reasoning users can select appropriate threshold \( \alpha \), which is a numeral value in \([0, 1]\), to create \( \alpha \)-cut \( R_\alpha \) of the fuzzy region \( R \):

\[
R_\alpha = \{ \mu_{R}(x,y) \geq \alpha \} 
\]  

(16)

\( R_\alpha \) denotes areas which suit the requirement of the task in certain suitable level.

3) Determining fuzzy membership function is a problem in application of the model. However, it can be solved by categorizing the vague language and providing every category analogous membership function model. This is a pivotal task for the application of field-based fuzzy spatial reasoning.

4) If the constraining factors have need of different weights in reasoning, the model can be reconstructed according to weighted multi-criteria evaluation approach.

7. Conclusions

Humans’ representation in natural language about geographic phenomena is always qualitative rather than quantitative. Qualitative spatial reasoning provides a nice approach which is considered to be closer to the representation. Future GIS commercial software should be equipped with artificial intelligent functions like qualitative spatial reasoning for more and more users, especially spatial decision-makers. This paper presents a field-based fuzzy qualitative spatial reasoning framework in the case of constraint satisfaction problems, in which humans’ vague language can be represented with field-based model in GIS. Two options available for choosing the membership functions for fuzzy sets are described since the choice of fuzzy membership function is crucial and strongly affects the results derived by a spatial reasoning process. Field-based fuzzy spatial reasoning operations in the case of constraints satisfaction problem are discussed. A simple example compares the approach with traditional spatial reasoning process. It is evident that the approach provides users more chance to choose suitable result as it provides the degree of satisfying the proposition proposed by the users.

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