Constructing Weighted Voronoi Diagrams Using Computer Programs

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Voronoi and Multiplicatively Weighted Voronoi Diagrams: A Brief Introduction and Definition

A classic way of regionalizing or allocating space around a predetermined set of points or generators is Voronoi tessellation. This method is based on finding the nearest generator for every point within the space. The resultant regionalization is known as a Voronoi diagram (VD). In a comprehensive presentation on the subject including an extensive review of the literature, Okabe et al., (2000) presented the ordinary VD as well many of its generalizations. In ordinary Voronoi diagrams all generator points are identical and each generator has the same weight. In a regionalized area by a VD borders are straight and the regions are contiguous.

In practical applications this assumption may not be appropriate. Rather, it is better to assume that generator points have different weights reflecting their variable properties; for example, the population size of a settlement, the number of functions in a shopping centre, the amount of emissions from a polluter, or the size of an atom in a crystal structure, and so forth. If weights are taken into account, then weighted Voronoi diagrams are generated. Depending on the variation methods they can be either multiplicatively, additively, or compoundly weighted diagrams (Okabe et al., 2000). In this introduction I focused on the multiplicatively weighted Voronoi diagram (MWVD).

Let S be a finite set of points in the Euclidean plane. Let p and q denote two points in the plane. Let the weights of the two points be $w(p)$ and $w(q)$. Let x be any point in the plane. The Euclidean distance between x and p is $d_e(x,p)$, and the weighted distance between x and p is $d_{mw}(x,p)$. The key to distinguishing between different types of weighted Voronoi is the definition of d_{mw}. Let *region (p)* denote the dominant region of point p; that is, p's influence region in S. In Figure 2-1, the weight of each point is labelled. The planar ordinary Voronoi diagram on the left of the figure can be defined as:

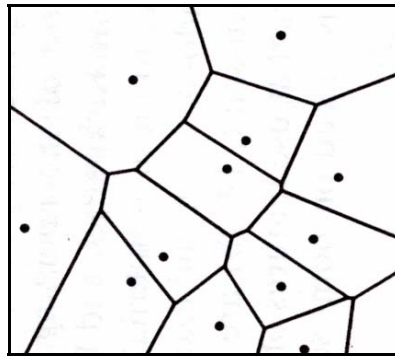$$region(p) = \{\, x \mid d_e(x,p) \le d_e(x,q),\ q \text{ in } S \,\}$$

The multiplicatively weighted Voronoi diagram (MW-Voronoi) on the right can be defined
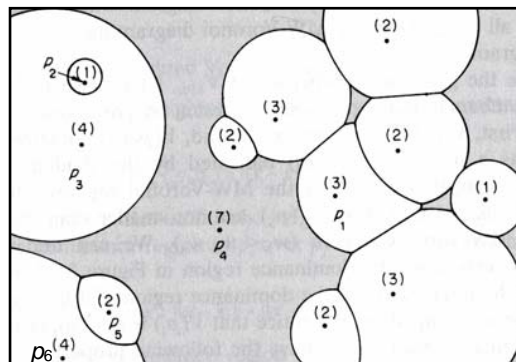
as:

$$region(p) = \{\ x|\ d_{mw}(x,p) \le d_{mw}(x,q),\ q\ in\ S\ \},$$
$$where\quad d_{mw}(x,p) = d_e(x,q)\ /\ w \qquad\qquad (MU\ Lan,\ 2004)$$

A Voronoi diagram is, thus, a method for dividing an area into regions or polygons so that all locations enclosed within a single polygon are closest to a designated point. A weighted

Voronoi Diagram

MW-Voronoi Diagram

Figure 1   Voronoi and multiplicatively weighted Voronoi diagrams.
(The numbers in parentheses represent weights associated with the generators)
Source: Okabe et al., 2000

Voronoi polygon can be utilized when points differ in size to draw polygons around them accordingly. Thus larger points will have larger polygons drawn while smaller points have smaller ones based on the distances between the points.

Constructing Multiplicatively Weighted Voronoi Diagrams

GIScience researchers want something beyond theoretical discussions and algorithm optimization. They would like to have some methods and implementation tools that allow them to actually use the model in research and teaching, the so-called off-the shelf- tools. Gambini

developed his computer program in FORTRAN IV (Gambini, 1966) to calculate equilibrium lines between points. Vincent also had his Network Analysis solution (Vincent and Daly, 1990). Recent software developments have made efforts to overcome this problem, including GAMBINI (Tiefelsdorf and Boots, 1997) and VORONOI (Gahegan and Lee, 2000), and WVD18 (Mu Lan, 2004).

The construction of weighted Voronoi diagrams is computationally more difficult than the Voronoi diagrams. The areas cannot be easily constructed within any standard GIS packages. Instead, specialist packages such as GAMBINI are available to complete the task. GAMBINI can construct weighted Voronoi diagrams that are proportional to a defined attribute. It is able to determine a sphere of influence around the generator points. In addition to being difficult to maintain, the polygons can not be properly built. Difficulties with the software arise more, when trying to create polygons for use in standard GIS packages (Pearce, 2000). Other packages are available that compute weighted Voronoi polygons and are likely to overcome some of these technical difficulties (Boots & South, 1997). WVD18 (Mu Lan, 2004) offered one of the few and newest specialist packages that provided precise computation MWVDs. Though it is not a GIS package, the created areas can easily be exported in shape file formats and then be added to GIS packages such as Arc/Info.

How to use WVD18 to construct Weighted Voronoi Diagrams?

Figure 2 shows a screenshot of the program. You can input points (generators) from the file menu in a shape file format or in a text file format. You can also input the X and Y of the points and their weight by clicking on the program window. If you input points from a text file, you need to follow an order for the X, Y, and the weights (see help menu in the program). When you create the Voronoi Diagrams, then, it is possible to save the file as a shape file.

Example: Figure 3 shows a sample text file for a set of schools including their weights, X, and Y, respectively. Figures 4 and 5, then, show a screenshot of the constructed Voronoi and weighted Voronoi areas for the schools using the text file.

To get the latest version of WVD18, and more information contact Dr. Mu Lan at: http://www.geog.uiuc.edu/people/mulan.html
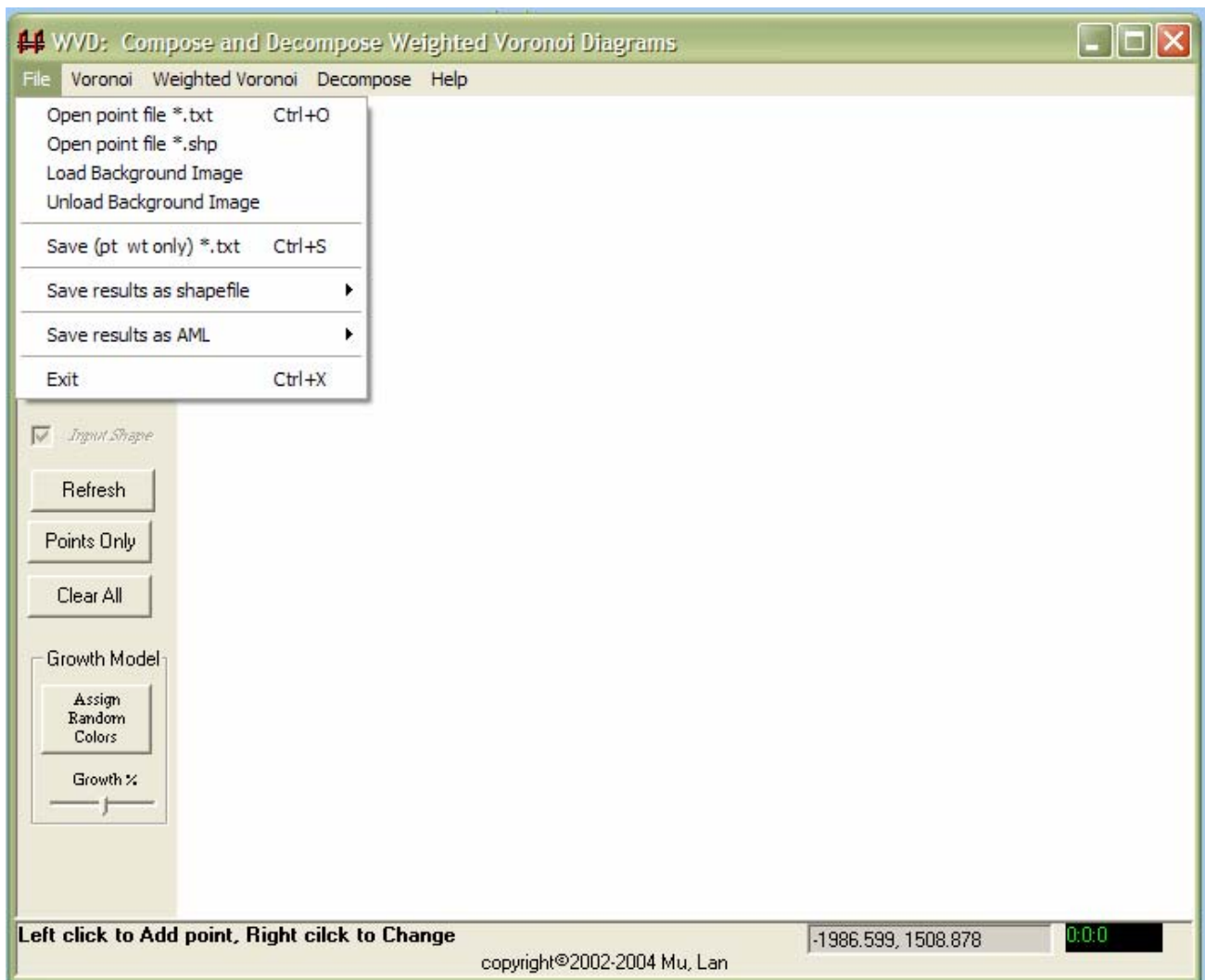
Figure 2   A screenshot of WVD18

Figure 3   A sample text file of a set of schools.
In each line, the numbers represent the weight (student population), X, and Y for each school, respecively.
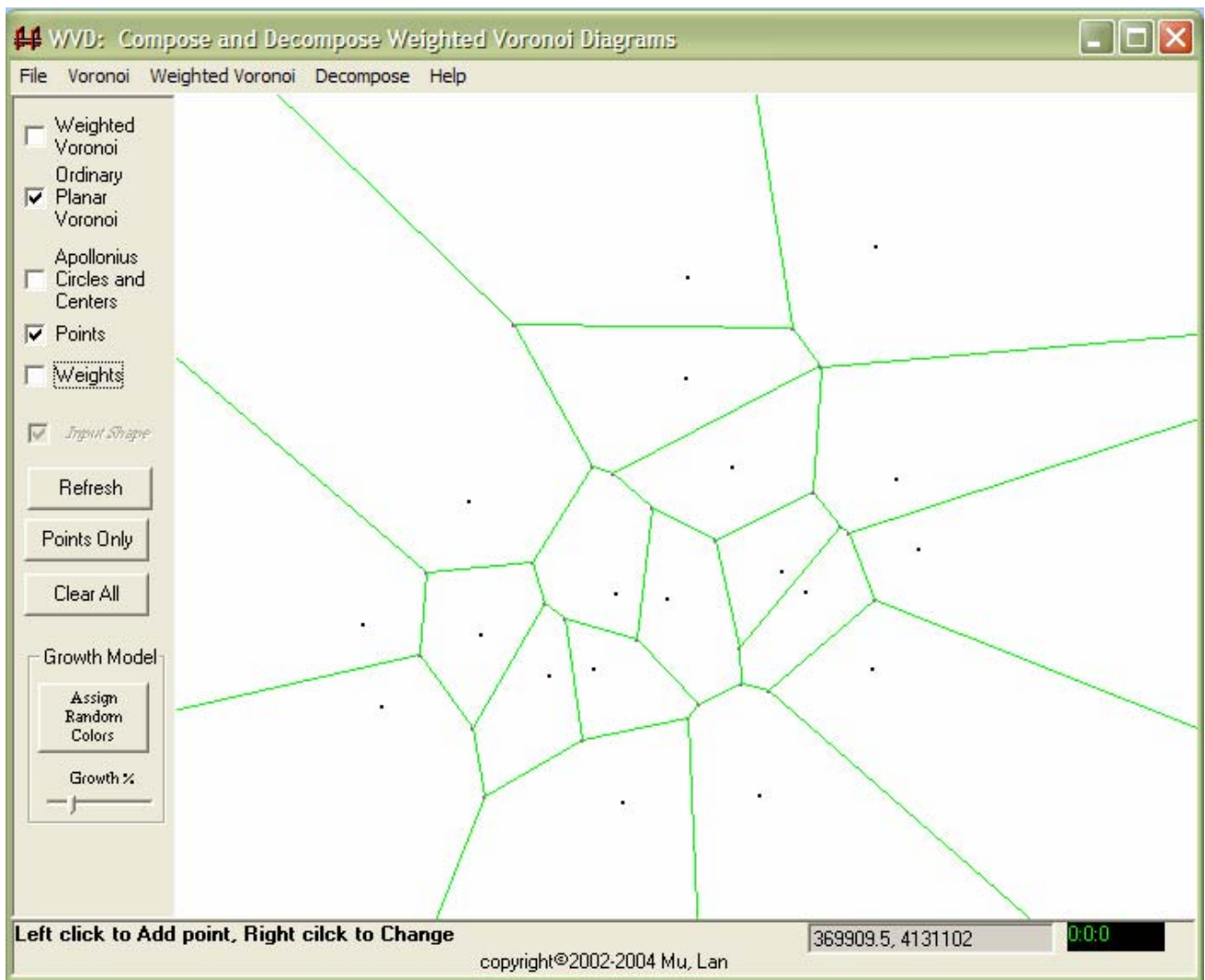
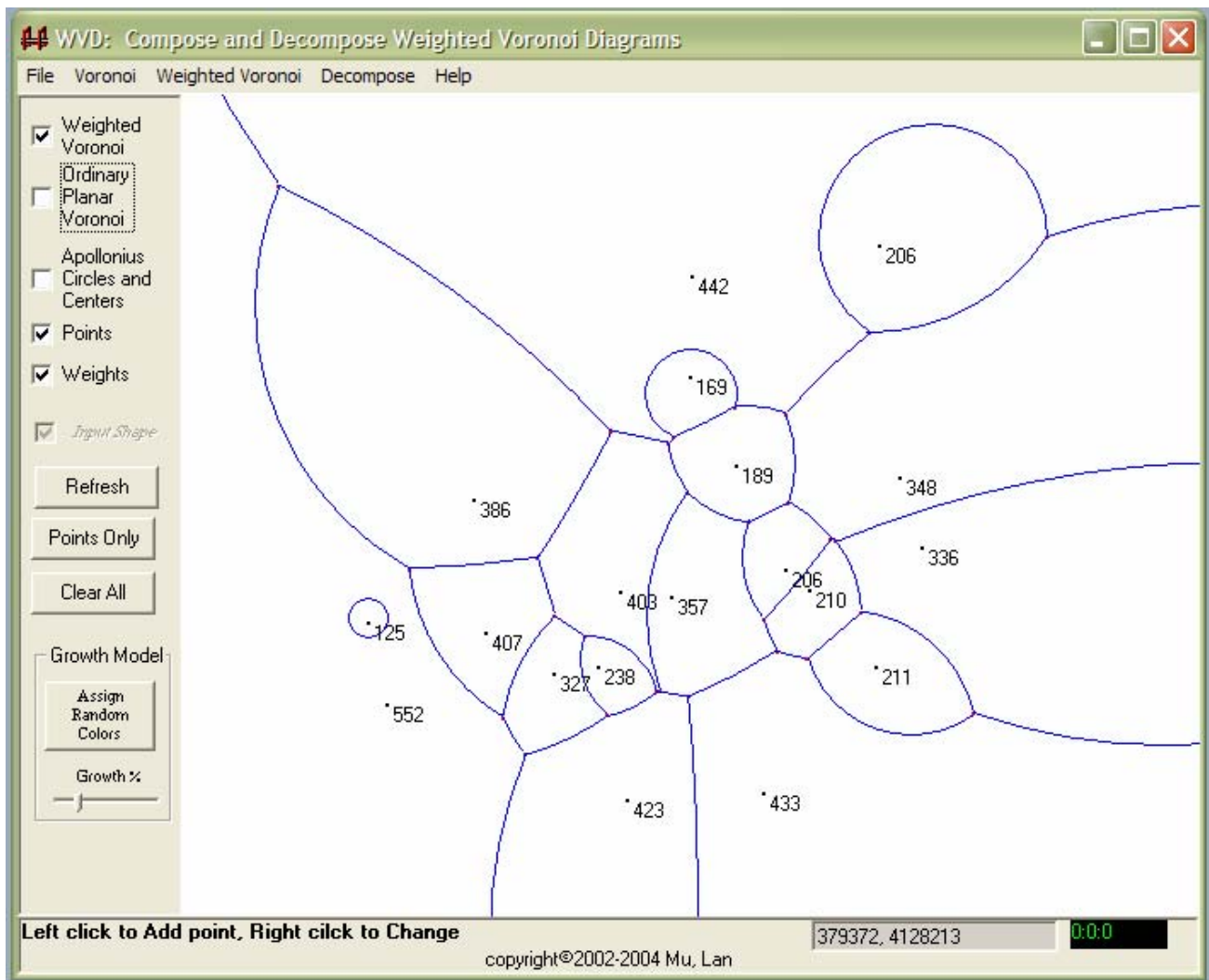Figure 4   The constructed Voronoi diagrams for the sample set of schools.

Figure 5   The constructed multiplicatively weighted Voronoi diagrams for
the sample set of schools.
The numbers represent the weights (student population) considered for each school.

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