How to do AHP analysis in Excel

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The Analytical Hierarchy Process - AHP

- AHP is one of the multiple criteria decision-making method that was originally developed by Prof. Thomas L. Saaty (1977).
- provides measures of judgement consistency
- derives priorities among criteria and alternatives
- simplifies preference ratings among decision criteria using pair wise comparisons

Using AHP

1. Decompose the decision-making problem into a **hierarchy**
2. Make **pair wise comparisons** and establish priorities among the elements in the hierarchy
3. **Synthesise** judgments (to obtain *the set of overall or weights* for achieving your goal)
4. Evaluate and check the **consistency** of judgements

The basic procedure is as follows:

1. **Develop the ratings for each decision alternative for each criterion by**

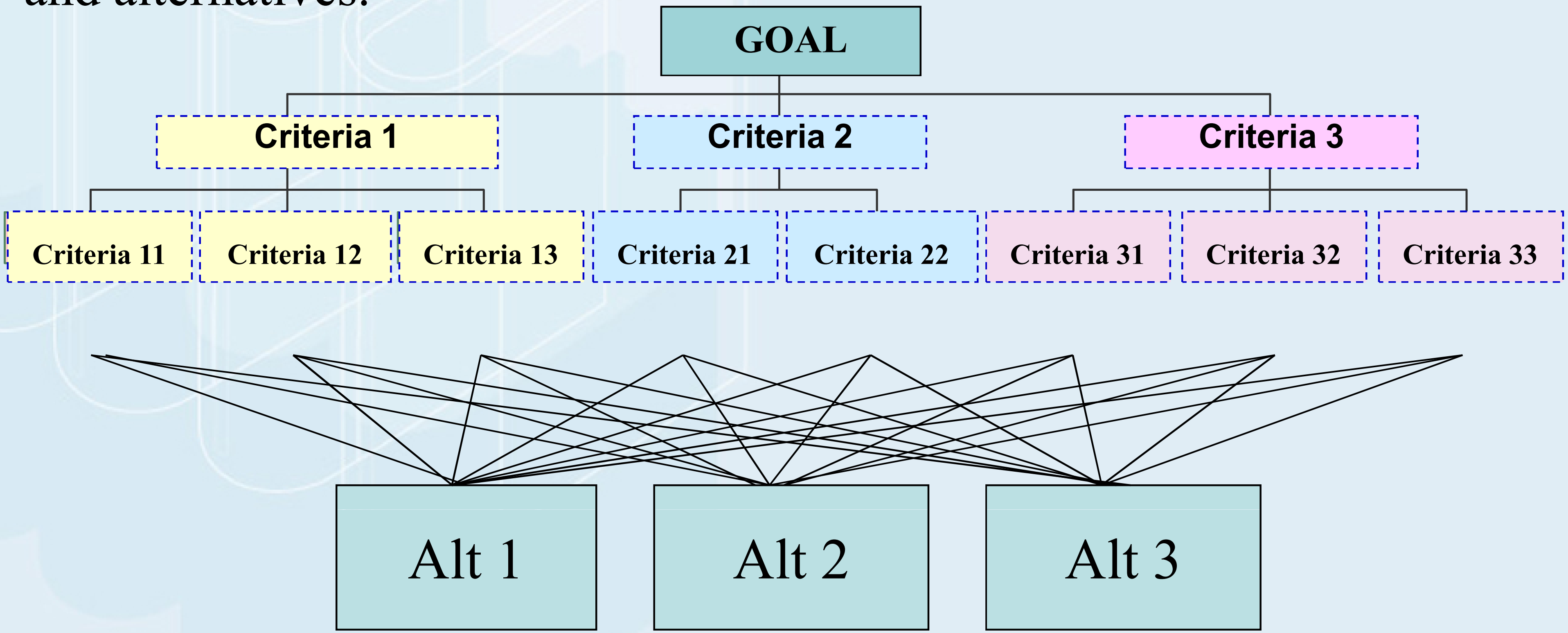
    - developing a pair wise comparison matrix for each criterion
    - normalizing the resulting matrix
    - averaging the values in each row to get the corresponding rating
    - calculating and checking the consistency ratio

2. **Develop the weights for the criteria by**
   - developing a pairwise comparison matrix for each criterion
   - normalizing the resulting matrix
   - averaging the values in each row to get the corresponding rating
   - calculating and checking the consistency ratio

3. **Calculate the weighted average rating for each decision alternative. Choose the one with the highest score.**

Structure the Hierarchy

Decompose the decision-making problem into a hierarchy of criteria and alternatives.



Level 1 is the goal of the analysis. Level 2 is multi-criteria that consist of several criterions, You can also add several other levels of sub-criteria. The last level is the alternative choices

The first step in the AHP procedure is to make pair wise comparisons between each criterion.

The example scale for comparison (Saaty & Vargas, 1991).

Scale	Degree of preference
1	Equal importance
3	Moderate importance of one factor over another
5	Strong or essential importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Values for inverse comparison

Results of the comparison (for each factors pair) were described in term of integer values from 1 (equal value) to 9 (extreme different) where higher number means the chosen factor is considered more important in greater degree than other factor being compared with.

Example

Table: Primary questionnaire design: effective criteria and pair wise comparison

Factor	Factor weighting score																	Factor
	More importance than								Equal	Less importance than								
C1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C2
C2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C3
C3	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C1

Table: Pair wise comparison matrix which holds the preference values

Criteria	C1	C2	C3
C1	1	4	5
C2	0.25	1	0.5
C3	0.2	2	1

=1/2

If the criteria in the column is preferred to the criteria in the row, then the inverse of the rating is given.

This table shows a simple comparison matrix of order 3 where 3 criteria C1, C2 and C3 are compared against each other.

Consider the following example:

Factor	Factor weighting score																	Factor
	More importance than								Equal	Less importance than								
C1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C2
C2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C3
C3	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C4
C4	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C5
C5	…	…	…	…	…	…	…	…	……	…	…	…	…	…	…	…	…	……

Start with the total cost criterion and generate the following data in a spreadsheet:

	A	B	C	D	E	F
1	Factor	C1	C2	C3	C4	C5
2	C1	1.00	7.00	3.00	1.00	1.00
3	C2		1.00	0.14	0.20	0.20
4	C3			1.00	1.00	1.00
5	C4				1.00	1.00
6	C5					1.00

Pair wise inputs

How to fill up the **upper triangular matrix** is using the following rules:

1. If the judgment value is on the **left** side of 1, we put the **actual judgment** value.
2. If the judgment value is on the **right** side of 1, we put the **reciprocal** value.

Making Comparison Matrix (How to make reciprocal matrix?)

To fill the **lower triangular matrix**, we use the reciprocal values of the upper diagonal. If $\mathbf{a_{ij}}$ is the element of row **i** column **j** of the matrix, then the lower diagonal is filled using this formula = $a_{ji} = \frac{1}{a_{ij}}$

	A	B	C	D	E	F
1	Factor	C1	C2	C3	C4	C5
2	C1	1.00	7.00	3.00	1.00	1.00
3	C2	=1/7	1.00	0.14	0.20	0.20
4	C3	=1/3	=1/0.14	1.00	1.00	1.00
5	C4	=1/1	=1/0.20	=1/1	1.00	1.00
6	C5	=1/1	=1/0.20	=1/1	=1/1	1.00

=1/C2

=1/D2

=1/E2

=1/F2

=1/E3

=1/F3

This slide shows how to analyze this paired comparisons

Step 1: Pair wise comparison

The criteria in the row is being compared to the criteria in the column.

Pair wise inputs

	A	B	C	D	E	F
1	Factor	C1	C2	C3	C4	C5
2	C1	1.00	7.00	3.00	1.00	1.00
3	C2	0.14	1.00	0.14	0.20	0.20
4	C3	0.33	7.00	1.00	1.00	1.00
5	C4	1.00	5.00	1.00	1.00	1.00
6	C5	1.00	5.00	1.00	1.00	1.00
7	Total	3.48	25.00	6.14	4.20	4.20

=Sum (B2:B6)

- Thus now we have complete comparison matrix
- The next step is to **normalize the matrix**. This is done by totaling the numbers in each column.

Step 2: Normalization

This step is to **normalize the matrix** by totaling the numbers in each column.

Each entry in the column is then divided by the column sum to yield its normalized score. The sum of each column is 1.

	A	B	C	D	E	F	G	H
6	C5	1.00	5.00	1.00	1.00	1.00		
7	Total	3.48	25.00	6.14	4.20	4.20		
8								
9	Factor	C1	C2	C3	C4	C5	Total	Average
10	C1	0.29	0.28	0.49	0.24	0.24	1.53	0.31
11	C2	0.04	0.04	0.02	0.05	0.05	0.20	0.04
12	C3	0.10	0.28	0.16	0.24	0.24	1.01	0.20
13	C4	0.29	0.20	0.16	0.24	0.24	1.13	0.23
14	C5	0.29	0.20	0.16	0.24	0.24	1.13	0.23

1=

=Sum (B10:F10)

=AVERAGE(G10/5)

=1

Normalized inputs (priority vector)

=(B6/B7)

=(C6/C7)

Highest average score

Step 3: Consistency analysis

Now, calculate the consistency ratio and check its value.

- The purpose for doing this is to make sure that the original preference ratings were consistent.

There are 3 steps to arrive at the consistency ratio:

1. Calculate the consistency measure.

2. Calculate the consistency index (CI).

$$\boxed{CI = \frac{\lambda max - n}{n - 1}}$$

3. Calculate the consistency ratio (CI/RI where RI is a random index).

$$\boxed{CR = CI / RI}$$

To calculate the consistency measure, we can take advantage of Excel's matrix multiplication function **=MMULT()**.

Consistency Ratio (CR)

	A	B	C	D	E	F	G	H	I
9	Factor	C1	C2	C3	C4	C5	Total	Average	Consistancy Measure
10	C1	0.29	0.28	0.49	0.24	0.24	1.53	0.31	5.37
11	C2	0.04	0.04	0.02	0.05	0.05	0.20	0.04	5.08
12	C3	0.10	0.28	0.16	0.24	0.24	1.01	0.20	5.10
13	C4	0.29	0.20	0.16	0.24	0.24	1.13	0.23	5.15
14	C5	0.29	0.20	0.16	0.24	0.24	1.13	0.23	5.15
15	Total	1.00	1.00	1.00	1.00	1.00		CI=	0.04
16								RI=	1.12
								C.Ratio	0.04

=MMULT(B2:F2,H10:H14)/H10

=MMULT(B3:F3,H10:H14)/H11

RI is provided by AHP (see slide 16)

=(AVERAGE(H10:H14)-5)/4

=I15/I16)

CR = CI / RI

Approximation of the Consistency Index

1. Multiply each column of the pair wise comparison matrix by the corresponding weight.
2. Divide of sum of the row entries by the corresponding weight.
3. Compute the average of the values from step 2, denote it by λmax .
4. The approximate

$$CI = \frac{\lambda max - n}{n - 1}$$

=(AVERAGE(H10:H14)-5)/4

Consistency Ratio (CR)

$$\boxed{CR = CI / RI}$$

- In practice, a CR of 0.1 or below is considered acceptable.
- Any higher value at any level indicate that the judgements warrant re-examination.

Consistency Index (CI)

- *reflects the consistency of one's judgement*

$$CI = \frac{\lambda max - n}{n - 1}$$

Random Index (RI)

- *the CI of a randomly-generated pair wise comparison matrix*

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.9	1.12	1.24	1.32	1.41	1.46	1.49

Notes: n = order of matrix
Random inconsistency indices for *n* = 10 (Saaty, 1980)

Summary

❖ With AHP, we can measure the degree of consistency; and if unacceptable, we can revise pair wise comparisons.

❖ If we are perfectly consistent, then the consistency measures will equal *n* and therefore, the CIs will be equal to zero and so will the consistency ratio.

❖ If this ratio is very large (Saaty suggests > 0.10), then we are not consistent enough and the best thing to do is go back and revise the comparisons.

❖ All of this work concludes the first step in the procedure. The next step is to use similar pair wise comparisons to determine the appropriate weights for each of the criteria.

❖ Now, continue for the other sub-criteria. You can easily do this by copying this sheet into other sheets and then simply changing the pair wise comparisons.

Remark

By now you have learned several introductory methods on Multi-Criteria Decision Making (MCDM) from the advantage of Excel's simple cross tabulation, using rank, and weighted score until AHP.

Widely Used AHP

- Cost/Benefit Analysis
- Strategic planning
- R&D priority setting and selection
- Technology choice
- Investment priority
- Priority for developing tourism
- Evaluation of for new telecommunications services
- Other evaluation of alternatives

The mathematics of AHP

(1) Normalization: “Behind the scene”

For a matrix of pair-wise elements: $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

1) sum the values in each column of the pair-wise matrix

$$C_{ij} = \sum_{i=1}^{n} C_{ij}$$

2) divide each element in the matrix by its column total to generate a normalized pair-wise matrix

$$X_{ij} = \frac{C_{ij}}{\sum_{i=1}^{n} C_{ij}} \qquad \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}$$

3) divide the sum of the normalized column of matrix by the number of criteria used (n) to generate weighted matrix

$$W_{ij} = \frac{\sum_{j=1}^{n} X_{ij}}{n} \qquad \begin{bmatrix} W_{11} \\ W_{12} \\ W_{13} \end{bmatrix}$$

(2) Consistency analysis : "Behind the scene"

Consistency Vector is calculated by multiplying the pair-wise matrix by the weights vector

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} * \begin{bmatrix} W_{11} \\ W_{21} \\ W_{31} \end{bmatrix} = \begin{bmatrix} Cv_{11} \\ Cv_{21} \\ Cv_{31} \end{bmatrix}$$

Then it is is accomplished by dividing the weighted sum vector with criterion weight

$$Cv_{11} = \frac{1}{W_{11}}\left[C_{11}W_{11} + \quad C_{12}W_{21} \quad C_{13}W_{31}\right]$$

$$Cv_{21} = \frac{1}{W_{21}}\left[C_{21}W_{11} + \quad C_{22}W_{21} \quad C_{23}W_{31}\right]$$

$$Cv_{31} = \frac{1}{W_{31}}\left[C_{31}W_{11} + \quad C_{32}W_{21} \quad C_{33}W_{31}\right]$$

λ is calculated by averaging the value of the Consistency Vector

$$\lambda = \sum_{i=1}^{n} Cv_{ij}$$

$$CI = \frac{\lambda - n}{n-1}$$

CI measures the deviation

$$C_r = \frac{CI}{RI}$$

$C_r < 0.10$

RANDOM INCONSISTENCY INDICES (RI) FOR $N = 10$

N	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.9	1.12	1.24	1.32	1.41	1.46	1.49

Source: Satty (1980).

Source: Haas, R. and Meixner, N.

References + Knowledges

Saaty, T.L. (1980). *The analytic hierarchy process*. McGraw-Hill, New York.

Saaty, T.L.,Vargas, L.G. (1991). *Prediction, Projection and Forecasting.* Kluwer Academic Publishers, Dordrecht, 251 pp.

Haas, R. and Meixner, N. (n.d.). *An Illustrated Guide to the Anlytic Hierarchy Process*. Institute of Marketing & Innovation, University of Natural Resources and Applied Life Sciences, Vienna [Available online] http://www.boku.ac.at/mi/

- ✓ Multi-Attribute Decision Analysis Approach: Qualitative Approach Analytic Hierarchy Process (AHP) – Expert Choice Exercise
- ✓ www.satecs.com – Some words on the Analytic Hierarchy Process (AHP) and the provided ArcGIS extension **'ext_ahp'**
- ✓ DECISION MODELING WITH MICROSOFT EXCEL: Multi-Objective Decision Making